

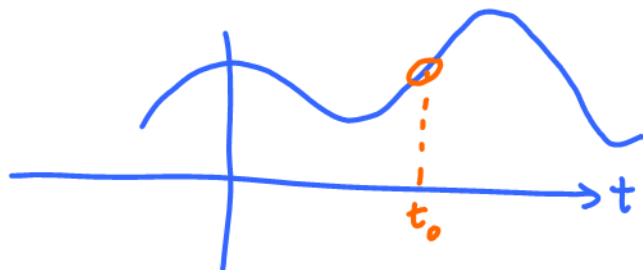
Q: What is the freq. of $x(t)$ around $t = t_0$?

Observation: If $\theta(t) = 2\pi f_0 t + \theta$

then $f(t_0) = f_0$

$$(f(t) = f_0, \forall t)$$

Now consider "any" function $\theta(t)$.



When we only focus on small interval around t_0 ,

$$\theta(t) \approx mt + c$$

↑
slope = $\theta'(t_0)$

Near $t = t_0$, $\theta(t) \approx (\theta'(t_0))t + c$

$$\cos(\theta(t)) \approx \cos(\theta'(t_0)t + c)$$

compare with

$$\cos(2\pi f_0 t + \phi)$$

$$\Rightarrow f(t_0) = \frac{\theta'(t_0)}{2\pi}$$

Conclusion: For generalized sinusoid $\cos(\theta(t))$, instantaneous freq. @ time t is given by

$$f(t) = \frac{\theta'(t)}{2\pi} = \frac{1}{2\pi} \frac{d}{dt} \theta(t)$$